

# PARI-GP Reference Card

(PARI-GP version 2.3.0)

Note: optional arguments are surrounded by braces {}.

## Starting & Stopping GP

to enter GP, just type its name: `gp`  
to exit GP, type `\q` or `quit`

## Help

describe function `?function`  
extended description `??keyword`  
list of relevant help topics `???pattern`

## Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`  
output from line `%n`  
separate multiple statements on line `;`  
extend statement on additional lines `\`  
extend statements on several lines `{seq1; seq2;}`  
comment `/* ... */`  
one-line comment, rest of line ignored `\\ ...`  
set default `d` to `val` `default({d}, {val}, flag)`  
mimic behaviour of GP 1.39 `default(compatible,3)`

## Metacommands

toggle timer on/off `#`  
print time for last result `##`  
print `%n` in raw format `\a n`  
print `%n` in pretty format `\b n`  
print defaults `\d`  
set debug level to `n` `\g n`  
set memory debug level to `n` `\gm n`  
enable/disable logfile `\l {filename}`  
print `%n` in pretty matrix format `\m`  
set output mode (raw, default, prettyprint) `\o n`  
set `n` significant digits `\p n`  
set `n` terms in series `\ps n`  
quit GP `\q`  
print the list of PARI types `\t`  
print the list of user-defined functions `\u`  
read file into GP `\r filename`  
write `%n` to file `\w n filename`

## GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`  
word completion `<TAB>`  
help menu window `M-\c`  
describe function `M-?`  
display TeX'd PARI manual `M-x gpman`  
set prompt string `M-\p`  
break line at column 100, insert `M-\`  
PARI metacommand `\letter` `M-\letter`

## Reserved Variable Names

$\pi = 3.14159\dots$  `Pi`  
Euler's constant `= .57721\dots` `Euler`  
square root of  $-1$  `I`  
big-oh notation `O`

## PARI Types & Input Formats

`t_INT`. Integers  $\pm n$   
`t_REAL`. Real Numbers  $\pm n.ddd$   
`t_INTMOD`. Integers modulo  $m$  `Mod(n, m)`  
`t_FRAC`. Rational Numbers  $n/m$   
`t_COMPLEX`. Complex Numbers  $x + y * I$   
`t_PADIC`.  $p$ -adic Numbers  $x + O(p^k)$   
`t_QUAD`. Quadratic Numbers  $x + y * \text{quadgen}(D)$   
`t_POLMOD`. Polynomials modulo  $g$  `Mod(f, g)`  
`t_POL`. Polynomials  $a * x^n + \dots + b$   
`t_SER`. Power Series  $f + O(x^k)$   
`t_QFI/t_QFR`. Imag/Real bin. quad. forms `Qfb(a, b, c, {d})`  
`t_RFRAC`. Rational Functions  $f/g$   
`t_VEC/t_COL`. Row/Column Vectors  $[x, y, z]$ ,  $[x, y, z]~$   
`t_MAT`. Matrices  $[x, y, z, t; u, v]$   
`t_LIST`. Lists `List([x, y, z])`  
`t_STR`. Strings `"aaa"`

## Standard Operators

basic operations `+, -, *, /, ^`  
`i=i+1, i=i-1, i=i*j, ...` `i++, i--, i*=j, ...`  
euclidean quotient, remainder `x\y, x\y, x%y, divrem(x, y)`  
shift `x` left or right `n` bits `x<<n, x>>n` or `shift(x, n)`  
comparison operators `<=, <, >=, >, ==, !=`  
boolean operators (or, and, not) `||, &&, !`  
sign of  $x = -1, 0, 1$  `sign(x)`  
maximum/minimum of  $x$  and  $y$  `max, min(x, y)`  
integer or real factorial of  $x$  `x!` or `factorial(x)`  
derivative of  $f$  w.r.t.  $x$  `f'`

## Conversions

**Change Objects**  
to vector, matrix, set, list, string `Col/Vec, Mat, Set, List, Str`  
create PARI object ( $x \bmod y$ ) `Mod(x, y)`  
make  $x$  a polynomial of  $v$  `Pol(x, {v})`  
as above, starting with constant term `Polrev(x, {v})`  
make  $x$  a power series of  $v$  `Ser(x, {v})`  
PARI type of object  $x$  `type(x, {t})`  
object  $x$  with precision  $n$  `prec(x, {n})`  
evaluate  $f$  replacing vars by their value `eval(f)`

**Select Pieces of an Object**  
length of  $x$  `#x` or `length(x)`  
 $n$ -th component of  $x$  `component(x, n)`  
 $n$ -th component of vector/list  $x$  `x[n]`  
 $(m, n)$ -th component of matrix  $x$  `x[m, n]`  
row  $m$  or column  $n$  of matrix  $x$  `x[m, ], x[, n]`  
numerator of  $x$  `numerator(x)`  
lowest denominator of  $x$  `denominator(x)`  
**Conjugates and Lifts**  
conjugate of a number  $x$  `conj(x)`  
conjugate vector of algebraic number  $x$  `conjvec(x)`  
norm of  $x$ , product with conjugate `norm(x)`  
square of  $L^2$  norm of vector  $x$  `norml2(x)`  
lift of  $x$  from Mods `lift, centerlift(x)`

## Random Numbers

random integer between 0 and  $N - 1$  `random({N})`  
get random seed `getrand()`  
set random seed to  $s$  `setrand(s)`

## Lists, Sets & Sorting

sort  $x$  by  $k$ th component `vecsort(x, {k}, {fl = 0})`  
**Sets** (= row vector of strings with strictly increasing entries)  
intersection of sets  $x$  and  $y$  `setintersect(x, y)`  
set of elements in  $x$  not belonging to  $y$  `setminus(x, y)`  
union of sets  $x$  and  $y$  `setunion(x, y)`  
look if  $y$  belongs to the set  $x$  `setsearch(x, y, flag)`  
**Lists**  
create empty list of maximal length  $n$  `listcreate(n)`  
delete all components of list  $l$  `listkill(l)`  
append  $x$  to list  $l$  `listput(l, x, {i})`  
insert  $x$  in list  $l$  at position  $i$  `listinsert(l, x, i)`  
sort the list  $l$  `listsort(l, flag)`

## Programming & User Functions

**Control Statements** ( $X$ : formal parameter in expression  $seq$ )  
eval.  $seq$  for  $a \leq X \leq b$  `for(X = a, b, seq)`  
eval.  $seq$  for  $X$  dividing  $n$  `fordiv(n, X, seq)`  
eval.  $seq$  for primes  $a \leq X \leq b$  `forprime(X = a, b, seq)`  
eval.  $seq$  for  $a \leq X \leq b$  stepping  $s$  `forstep(X = a, b, s, seq)`  
multivariable for `forvec(X = v, seq)`  
if  $a \neq 0$ , evaluate  $seq_1$ , else  $seq_2$  `if(a, {seq1}, {seq2})`  
evaluate  $seq$  until  $a \neq 0$  `until(a, seq)`  
while  $a \neq 0$ , evaluate  $seq$  `while(a, seq)`  
exit  $n$  innermost enclosing loops `break({n})`  
start new iteration of  $n$ th enclosing loop `next({n})`  
return  $x$  from current subroutine `return(x)`  
error recovery (try  $seq_1$ ) `trap({err}, {seq2}, {seq1})`

## Input/Output

prettyprint args with/without newline `printp(), printp1()`  
print args with/without newline `print(), print1()`  
read a string from keyboard `input()`  
reorder priority of variables  $x, y, z$  `reorder({[x, y, z]})`  
output  $args$  in TeX format `printtex(args)`  
write  $args$  to file `write, write1, writetex(file, args)`  
read file into GP `read({file})`

## Interface with User and System

allocates a new stack of  $s$  bytes `allocatemem({s})`  
execute system command  $a$  `system(a)`  
as above, feed result to GP `extern(a)`  
install function from library `install(f, code, {gpf}, {lib})`  
alias  $old$  to  $new$  `alias(new, old)`  
new name of function  $f$  in GP 2.0 `whatnow(f)`

## User Defined Functions

`name(formal vars) = local(local vars); seq`  
`struct.member = seq`  
kill value of variable or function  $x$  `kill(x)`  
declare global variables `global(x, ...)`

## Iterations, Sums & Products

numerical integration `intnum(X = a, b, expr, flag)`  
sum  $expr$  over divisors of  $n$  `sumdiv(n, X, expr)`  
sum  $X = a$  to  $X = b$ , initialized at  $x$  `sum(X = a, b, expr, {x})`  
sum of series  $expr$  `suminf(X = a, expr)`  
sum of alternating/positive series `sumalt, sumpos`  
product  $a \leq X \leq b$ , initialized at  $x$  `prod(X = a, b, expr, {x})`  
product over primes  $a \leq X \leq b$  `prodeuler(X = a, b, expr)`  
infinite product  $a \leq X \leq \infty$  `prodinf(X = a, expr)`  
real root of  $expr$  between  $a$  and  $b$  `solve(X = a, b, expr)`

## Vectors & Matrices

dimensions of matrix $x$	<code>matsize(x)</code>
concatenation of $x$ and $y$	<code>concat(x, {y})</code>
extract components of $x$	<code>vecextract(x, y, {z})</code>
transpose of vector or matrix $x$	<code>mattranspose(x)</code> or <code>x-matadjoint(x)</code>
adjoint of the matrix $x$	<code>mateigen(x)</code>
eigenvectors of matrix $x$	<code>charpoly(x, {v}, flag)</code>
characteristic polynomial of $x$	<code>minpoly(x, {v})</code>
minimal polynomial of $x$	<code>trace(x)</code>
trace of matrix $x$	

### Constructors & Special Matrices

row vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vector(n, {i}, {expr})</code>
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vectorv(n, {i}, {expr})</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix(m, n, {i}, {j}, {expr})</code>
diagonal matrix whose diag. is $x$	<code>matdiagonal(x)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix $x$	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$	<code>matpascal(n - 1)</code>
companion matrix to polynomial $x$	<code>matcompanion(x)</code>

### Gaussian elimination

determinant of matrix $x$	<code>matdet(x, flag)</code>
kernel of matrix $x$	<code>matker(x, flag)</code>
intersection of column spaces of $x$ and $y$	<code>matintersect(x, y)</code>
solve $M * X = B$ ( $M$ invertible)	<code>matsolve(M, B)</code>
as solve, modulo $D$ (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix $x$	<code>matimage(x)</code>
supplement columns of $x$ to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix $x$	<code>matrank(x)</code>

## Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of $x$ where $d$ is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
elementary divisors of $x$	<code>matsnf(x)</code>
LLL-algorithm applied to columns of $x$	<code>qflll(x, flag)</code>
like <code>qflll</code> , $x$ is Gram matrix of lattice	<code>qflllgram(x, flag)</code>
LLL-reduced basis for kernel of $x$	<code>matkerint(x)</code>
$\mathbf{Z}$ -lattice $\longleftrightarrow$ $\mathbf{Q}$ -vector space	<code>matrixqz(x, p)</code>
signature of quad form $t^y * x * y$	<code>qf sign(x)</code>
decomp into squares of $t^y * x * y$	<code>qfgaussred(x)</code>
find up to $m$ sols of $t^y * x * y \leq b$	<code>qfminim(x, b, m)</code>
$v, v[i] :=$ number of sols of $t^y * x * y = i$	<code>qfrep(x, B, flag)</code>
eigenvals/eigenvecs for real symmetric $x$	<code>qfjacobi(x)</code>

## Formal & p-adic Series

truncate power series or $p$ -adic number	<code>truncate(x)</code>
valuation of $x$ at $p$	<code>valuation(x, p)</code>

### Dirichlet and Power Series

Taylor expansion around 0 of $f$ w.r.t. $x$	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(x, y)</code>
$f = \sum a_k * t^k$ from $\sum (a_k/k!) * t^k$	<code>serlaplace(f)</code>
reverse power series $F$ so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product ( $b$ terms)	<code>direuler(p = a, b, expr)</code>

### p-adic Functions

Teichmuller character of $x$	<code>teichmuller(x)</code>
Newton polygon of $f$ for prime $p$	<code>newtonpoly(f, p)</code>

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## Polynomials & Rational Functions

degree of $f$	<code>poldegree(f)</code>
coefficient of degree $n$ of $f$	<code>polcoeff(f, n)</code>
round coeffs of $f$ to nearest integer	<code>round(f, {&amp;e})</code>
gcd of coefficients of $f$	<code>content(f)</code>
replace $x$ by $y$ in $f$	<code>subst(f, x, y)</code>
discriminant of polynomial $f$	<code>poldisc(f)</code>
resultant of $f$ and $g$	<code>polresultant(f, g, flag)</code>
as above, give $[u, v, d], xu + yv = d$	<code>bezoutres(x, y)</code>
derivative of $f$ w.r.t. $x$	<code>deriv(f, x)</code>
formal integral of $f$ w.r.t. $x$	<code>intformal(f, x)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
interpol. pol. eval. at $a$	<code>polinterpolate(X, {Y}, {a}, {&amp;e})</code>
initialize $t$ for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue(t, a, {sol})</code>

### Roots and Factorization

number of real roots of $f, a < x \leq b$	<code>polsturm(f, {a}, {b})</code>
complex roots of $f$	<code>polroots(f)</code>
symmetric powers of roots of $f$ up to $n$	<code>polsym(f, n)</code>
roots of $f$ mod $p$	<code>polrootsmod(f, p, flag)</code>
factor $f$	<code>factor(f, {lim})</code>
factorization of $f$ mod $p$	<code>factormod(f, p, flag)</code>
factorization of $f$ over $\mathbf{F}_{p^a}$	<code>factorff(f, p, a)</code>
$p$ -adic fact. of $f$ to prec. $r$	<code>factorpadic(f, p, r, flag)</code>
$p$ -adic roots of $f$ to prec. $r$	<code>polrootspadic(f, p, r)</code>
$p$ -adic root of $f$ cong. to $a$ mod $p$	<code>padicappr(f, a)</code>
Newton polygon of $f$ for prime $p$	<code>newtonpoly(f, p)</code>

### Special Polynomials

$n$ th cyclotomic polynomial in var. $v$	<code>polcyclo(n, {v})</code>
$d$ -th degree subfield of $\mathbf{Q}(\zeta_n)$	<code>polsubcyclo(n, d, {v})</code>
$n$ -th Legendre polynomial	<code>pollegendre(n)</code>
$n$ -th Tchebicheff polynomial	<code>poltchebi(n)</code>
Zagier's polynomial of index $n, m$	<code>polzagier(n, m)</code>

## Transcendental Functions

real, imaginary part of $x$	<code>real(x), imag(x)</code>
absolute value, argument of $x$	<code>abs(x), arg(x)</code>
square/ $n$ th root of $x$	<code>sqr(x), sqrtn(x, n, &amp;z)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential of $x$	<code>exp(x)</code>
natural log of $x$	<code>ln(x) or log(x)</code>
gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$	<code>gamma(x)</code>
logarithm of gamma function	<code>lngamma(x)</code>
$\psi(x) = \Gamma'(x)/\Gamma(x)$	<code>psi(x)</code>
incomplete gamma function ( $y = \Gamma(s)$ )	<code>incgam(s, x, {y})</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of $x$	<code>dilog(x)</code>
$m$ th polylogarithm of $x$	<code>polylog(m, x, flag)</code>
$U$ -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
$J$ -Bessel function $J_{n+1/2}(x)$	<code>besseljh(n, x)</code>
$K$ -Bessel function of index $nu$	<code>besselk(nu, x)</code>

## Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
give bit number $n$ of integer $x$	<code>bittest(x, n)</code>
ceiling of $x$	<code>ceil(x)</code>
floor of $x$	<code>floor(x)</code>
fractional part of $x$	<code>frac(x)</code>
round $x$ to nearest integer	<code>round(x, {&amp;e})</code>
truncate $x$	<code>truncate(x, {&amp;e})</code>
gcd/LCM of $x$ and $y$	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

### Primes and Factorization

add primes in $v$ to the prime table	<code>addprimes(v)</code>
the $n$ th prime	<code>prime(n)</code>
vector of first $n$ primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>preprime(x)</code>
factorization of $x$	<code>factor(x, {lim})</code>
reconstruct $x$ from its factorization	<code>factorback(fa, {nf})</code>

### Divisors

number of distinct prime divisors	<code>omega(x)</code>
number of prime divisors with mult	<code>bigomega(x)</code>
number of divisors of $x$	<code>numdiv(x)</code>
row vector of divisors of $x$	<code>divisors(x)</code>
sum of ( $k$ -th powers of) divisors of $x$	<code>sigma(x, {k})</code>

### Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number $B_n$ as real	<code>bernreal(n)</code>
Bernoulli vector $B_0, B_2, \dots, B_{2n}$	<code>bernvec(n)</code>
$n$ th Fibonacci number	<code>fibonacci(n)</code>
number of partitions of $n$	<code>numbpart(n)</code>
Euler $\phi$ -function	<code>eulerphi(x)</code>
Möbius $\mu$ -function	<code>moebius(x)</code>
Hilbert symbol of $x$ and $y$ (at $p$ )	<code>hilbert(x, y, {p})</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>

### Miscellaneous

integer or real factorial of $x$	<code>x!</code> or <code>fact(x)</code>
integer square root of $x$	<code>sqr(x)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal $u, v$ so $xu + yv = \gcd(x, y)$	<code>bezout(x, y)</code>
multiplicative order of $x$ (intmod) ( $i=0$ )	<code>znorder(x, {o})</code>
primitive root mod prime power $q$	<code>znprimroot(q)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
continued fraction of $x$	<code>contfrac(x, {b}, {lmax})</code>
last convergent of continued fraction $x$	<code>contfracpnqn(x)</code>
best rational approximation to $x$	<code>bestappr(x, k)</code>

## True-False Tests

is $x$ the disc. of a quadratic field?	<code>isfundamental(x)</code>
is $x$ a prime?	<code>isprime(x)</code>
is $x$ a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is $x$ square-free?	<code>issquarefree(x)</code>
is $x$ a square?	<code>Z_issquare(x, {&amp;n})</code>
is $pol$ irreducible?	<code>polisirreducible(pol)</code>

Based on an earlier version by Joseph H. Silverman  
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# PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

## Elliptic Curves

Elliptic curve initially given by 5-tuple  $E = [a_1, a_2, a_3, a_4, a_6]$ . Points are  $[x, y]$ , the origin is  $[0]$ .

Initialize elliptic struct.  $ell$ , i.e create `ellinit( $E, flag$ )`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$ . This data can be recovered by typing `ell.a1, ..., ell.j`. If  $fl$  omitted, also  $E$  defined over **R**

$x$ -coords. of points of order 2	<code>ell.roots</code>
real and complex periods	<code>ell.omega</code>
associated quasi-periods	<code>ell.eta</code>
volume of complex lattice	<code>ell.area</code>

$E$  defined over  $\mathbf{Q}_p$ ,  $|j|_p > 1$

$x$ -coord. of unit 2 torsion point	<code>ell.roots</code>
Tate's $[u^2, u, q]$	<code>ell.tate</code>
Mestre's $w$	<code>ell.w</code>

change curve  $E$  using  $v = [u, r, s, t]$  `ellchangecurve( $ell, v$ )`

change point  $z$  using  $v = [u, r, s, t]$  `ellchangept( $z, v$ )`

cond, min mod, Tamagawa num  $[N, v, c]$  `ellglobalred( $ell$ )`

Kodaira type of  $p$  fiber of  $E$  `elllocalred( $ell, p$ )`

add points  $z_1 + z_2$  `elladd( $ell, z_1, z_2$ )`

subtract points  $z_1 - z_2$  `ellsub( $ell, z_1, z_2$ )`

compute  $n \cdot z$  `ellpow( $ell, z, n$ )`

check if  $z$  is on  $E$  `ellisoncurve( $ell, z$ )`

order of torsion point  $z$  `ellorder( $ell, z$ )`

torsion subgroup with generators `elltors( $ell$ )`

$y$ -coordinates of point(s) for  $x$  `ellordinate( $ell, x$ )`

canonical bilinear form taken at  $z_1, z_2$  `ellbil( $ell, z_1, z_2$ )`

canonical height of  $z$  `ellheight( $ell, z, flag$ )`

height regulator matrix for pts in  $x$  `ellheightmatrix( $ell, x$ )`

$p$ th coeff  $a_p$  of  $L$ -function,  $p$  prime `ellap( $ell, p$ )`

$k$ th coeff  $a_k$  of  $L$ -function `ellak( $ell, k$ )`

vector of first  $n$   $a_k$ 's in  $L$ -function `ellan( $ell, n$ )`

$L(E, s)$ , set  $A \approx 1$  `elllseries( $ell, s, \{A\}$ )`

root number for  $L(E, \cdot)$  at  $p$  `ellrootno( $ell, \{p\}$ )`

modular parametrization of  $E$  `elltaniyama( $ell$ )`

point  $[\wp(z), \wp'(z)]$  corresp. to  $z$  `ellztopoint( $ell, z$ )`

complex  $z$  such that  $p = [\wp(z), \wp'(z)]$  `ellpointtoz( $ell, p$ )`

## Elliptic & Modular Functions

arithmetic-geometric mean `agm( $x, y$ )`

elliptic  $j$ -function  $1/q + 744 + \dots$  `ellj( $x$ )`

Weierstrass  $\sigma$  function `ellsigma( $ell, z, flag$ )`

Weierstrass  $\wp$  function `ellwp( $ell, \{z\}, flag$ )`

Weierstrass  $\zeta$  function `ellzeta( $ell, z$ )`

modified Dedekind  $\eta$  func.  $\prod(1 - q^n)$  `eta( $x, flag$ )`

Jacobi sine theta function `theta( $q, z$ )`

$k$ -th derivative at  $z=0$  of `thetanulk( $q, k$ )`

Weber's  $f$  functions `weber( $x, flag$ )`

Riemann's zeta  $\zeta(s) = \sum n^{-s}$  `zeta( $s$ )`

## Graphic Functions

crude graph of  $expr$  between  $a$  and  $b$  `plot( $X = a, b, expr$ )`

**High-resolution plot** (immediate plot) `plot(X = a, b, expr, flag, \{n\})`

plot  $expr$  between  $a$  and  $b$  `plot(X = a, b, expr, flag, \{n\})`

plot points given by lists  $lx, ly$  `plotdraw( $lx, ly, flag$ )`

terminal dimensions `plotsizes()`

### Rectwindow functions

init window  $w$ , with size  $x, y$  `plotinit( $w, x, y$ )`

erase window  $w$  `plotkill( $w$ )`

copy  $w$  to  $w_2$  with offset  $(dx, dy)$  `plotcopy( $w, w_2, dx, dy$ )`

scale coordinates in  $w$  `plotscale( $w, x_1, x_2, y_1, y_2$ )`

`plot` in  $w$  `plotrecth( $w, X = a, b, expr, flag, \{n\}$ )`

`plotdraw` in  $w$  `plotrecthdraw( $w, data, flag$ )`

draw window  $w_1$  at  $(x_1, y_1), \dots$  `plotdraw([[w1, x1, y1], ...])`

### Low-level Rectwindow Functions

set current drawing color in  $w$  to  $c$  `plotcolor( $w, c$ )`

current position of cursor in  $w$  `plotcursor( $w$ )`

write  $s$  at cursor's position `plotstring( $w, s$ )`

move cursor to  $(x, y)$  `plotmove( $w, x, y$ )`

move cursor to  $(x + dx, y + dy)$  `plotrmove( $w, dx, dy$ )`

draw a box to  $(x_2, y_2)$  `plotbox( $w, x_2, y_2$ )`

draw a box to  $(x + dx, y + dy)$  `plotrbox( $w, dx, dy$ )`

draw polygon `plotlines( $w, lx, ly, flag$ )`

draw points `plotpoints( $w, lx, ly$ )`

draw line to  $(x + dx, y + dy)$  `plotrline( $w, dx, dy$ )`

draw point  $(x + dx, y + dy)$  `plotrpoint( $w, dx, dy$ )`

### Postscript Functions

as `plot` `psplot(X = a, b, expr, flag, \{n\})`

as `plotdraw` `psplotdraw( $lx, ly, flag$ )`

as `plotdraw` `psdraw([[w1, x1, y1], ...])`

## Binary Quadratic Forms

create  $ax^2 + bxy + cy^2$  (distance  $d$ ) `qfb( $a, b, c, \{d\}$ )`

reduce  $x$  ( $s = \sqrt{D}$ ,  $l = \lfloor s \rfloor$ ) `qfbred( $x, flag, \{D\}, \{l\}, \{s\}$ )`

composition of forms  $x*y$  or `qfbnucomp( $x, y, l$ )`

$n$ -th power of form  $x^n$  or `qfbnpow( $x, n$ )`

composition without reduction `qfbcompraw( $x, y$ )`

$n$ -th power without reduction `qfbpowraw( $x, n$ )`

prime form of disc.  $x$  above prime  $p$  `qfbprimeform( $x, p$ )`

class number of disc.  $x$  `qfbclassno( $x$ )`

Hurwitz class number of disc.  $x$  `qfbhclassno( $x$ )`

## Quadratic Fields

quadratic number  $\omega = \sqrt{x}$  or  $(1 + \sqrt{x})/2$  `quadgen( $x$ )`

minimal polynomial of  $\omega$  `quadpoly( $x$ )`

discriminant of  $\mathbf{Q}(\sqrt{D})$  `quaddisc( $x$ )`

regulator of real quadratic field `quadregulator( $x$ )`

fundamental unit in real  $\mathbf{Q}(x)$  `quadunit( $x$ )`

class group of  $\mathbf{Q}(\sqrt{D})$  `quadclassunit( $D, flag, \{t\}$ )`

Hilbert class field of  $\mathbf{Q}(\sqrt{D})$  `quadhilbert( $D, flag$ )`

ray class field modulo  $f$  of  $\mathbf{Q}(\sqrt{D})$  `quadray( $D, f, flag$ )`

## General Number Fields: Initializations

A number field  $K$  is given by a monic irreducible  $f \in \mathbf{Z}[X]$ .

init number field structure  $nf$  `nfinit( $f, flag$ )`

### nf members:

polynomial defining  $nf$ ,  $f(\theta) = 0$  `nf.pol`

number of real/complex places `nf.r1, nf.r2`

discriminant of  $nf$  `nf.disc`

$T_2$  matrix `nf.t2`

vector of roots of  $f$  `nf.roots`

integral basis of  $\mathbf{Z}_K$  as powers of  $\theta$  `nf.zk`

different `nf.diff`

codifferent `nf.codiff`

recompute  $nf$  using current precision `nfnewprec( $nf$ )`

init relative  $rmf$  given by  $g = 0$  over  $K$  `rmfinit( $nf, g$ )`

init  $bnf$  structure `bnfinit( $f, flag$ )`

**bnf members:** same as  $nf$ , plus

underlying  $nf$  `bnf.nf`

classgroup `bnf.clgp`

regulator `bnf.reg`

fundamental units `bnf.fu`

torsion units `bnf.tu`

$[tu, fu]$  `bnf.tufu`

compute a  $bnf$  from small  $bnf$  `bnfmake( $sbnf$ )`

add  $S$ -class group and units, yield  $bnf$  `bnfsunit( $nf, S$ )`

init class field structure  $bnr$  `bnrinit( $bnf, m, flag$ )`

**bnr members:** same as  $bnf$ , plus

underlying  $bnf$  `bnr.bnf`

structure of  $(\mathbf{Z}_K/m)^*$  `bnr.zkst`

## Simple Arithmetic Invariants (nf)

Elements are rational numbers, polynomials, polmods, or column vectors (on integral basis  $nf.zk$ ).  
integral basis of field def. by  $f = 0$  `nfbasis(f)`  
field discriminant of field  $f = 0$  `nfdisc(f)`  
reverse polmod  $a = A(X) \bmod T(X)$  `modreverse(a)`  
Galois group of field  $f = 0$ ,  $\deg f \leq 11$  `polgalois(f)`  
smallest poly defining  $f = 0$  `polredabs(f, flag)`  
small polys defining subfields of  $f = 0$  `polred(f, flag, {p})`  
small polys defining suborders of  $f = 0$  `polredord(f)`  
poly of degree  $\leq k$  with root  $x \in \mathbf{C}$  `algdep(x, k)`  
small linear rel. on coords of vector  $x$  `linddep(x)`  
are fields  $f = 0$  and  $g = 0$  isomorphic? `nfisom(f, g)`  
is field  $f = 0$  a subfield of  $g = 0$ ? `nfisincl(f, g)`  
compositum of  $f = 0$ ,  $g = 0$  `polcompositum(f, g, flag)`  
basic element operations (prefix `nfelt`):

(`nfelt`)`mul`, `pow`, `div`, `divauc`, `mod`, `divrem`, `val`  
express  $x$  on integer basis `nfalgtobasis(nf, x)`  
express element  $x$  as a polmod `nfbasistoalg(nf, x)`  
quadratic Hilbert symbol (at  $p$ ) `nfhilbert(nf, a, b, {p})`  
roots of  $g$  belonging to  $nf$  `nfroots({nf}, g)`  
factor  $g$  in  $nf$  `nfactor(nf, g)`  
factor  $g$  mod prime  $pr$  in  $nf$  `nfactormod(nf, g, pr)`  
number of roots of unity in  $nf$  `nfrootsof1(nf)`  
conjugates of a root  $\theta$  of  $nf$  `nfgaloisconj(nf, flag)`  
apply Galois automorphism  $s$  to  $x$  `nfgaloisapply(nf, s, x)`  
subfields (of degree  $d$ ) of  $nf$  `nfsubfields(nf, {d})`

### Dedekind Zeta Function $\zeta_K$

$\zeta_K$  as Dirichlet series,  $N(I) < b$  `dirzetak(nf, b)`  
init  $nfz$  for field  $f = 0$  `zetakinit(f)`  
compute  $\zeta_K(s)$  `zetak(nfz, s, flag)`  
Artin root number of  $K$  `bnrrootnumber(bnr, chi, flag)`

## Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$  usually  $bnr$ ,  $subgp$  or  $bnf$ ,  $module$ ,  $\{subgp\}$   
remove GRH assumption from  $bnf$  `bnfcertify(bnf)`  
expo. of ideal  $x$  on class gp `bnfisprincipal(bnf, x, flag)`  
expo. of ideal  $x$  on ray class gp `bnrisprincipal(bnr, x, flag)`  
expo. of  $x$  on fund. units `bnfisunit(bnf, x)`  
as above for  $S$ -units `bnfissunit(bnfs, x)`  
fundamental units of  $bnf$  `bnfunit(bnf)`  
signs of real embeddings of  $bnf.fu$  `bnfsignunit(bnf)`

### Class Field Theory

ray class group structure for mod.  $m$  `bnrclass(bnf, m, flag)`  
ray class number for mod.  $m$  `bnrclassno(bnf, m)`  
discriminant of class field ext `bnrdisc(a1, {a2}, {a3})`  
ray class numbers,  $l$  list of mods `bnrclassnolist(bnf, l)`  
discriminants of class fields `bnrdisclist(bnf, l, {arch}, flag)`  
decode output from `bnrdisclst` `bnfdecodemodule(nf, fa)`  
is modulus the conductor? `bnrisconductor(a1, {a2}, {a3})`  
conductor of character  $chi$  `bnrconductorofchar(bnr, chi)`  
conductor of extension `bnrconductor(a1, {a2}, {a3}, flag)`  
conductor of extension def. by  $g$  `rnfconductor(bnf, g)`  
Artin group of ext. def'd by  $g$  `rnfnormgroup(bnr, g)`  
subgroups of  $bnr$ , index  $\leq b$  `subgrouplist(bnr, b, flag)`  
rel. eq. for class field def'd by  $sub$  `rnfkummer(bnr, sub, {d})`  
same, using Stark units (real field) `bnrstark(bnr, sub, flag)`

## PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

### Ideals

Ideals are elements, primes, or matrix of generators in HNF.  
is  $id$  an ideal in  $nf$ ? `nfisideal(nf, id)`  
is  $x$  principal in  $bnf$ ? `bnfisprincipal(bnf, x)`  
principal ideal generated by  $x$  `idealprincipal(nf, x)`  
principal idele generated by  $x$  `ideleprincipal(nf, x)`  
give  $[a, b]$ , s.t.  $a\mathbf{Z}_K + b\mathbf{Z}_K = x$  `idealtwoelt(nf, x, {a})`  
put ideal  $a$  ( $a\mathbf{Z}_K + b\mathbf{Z}_K$ ) in HNF form `idealhnf(nf, a, {b})`  
norm of ideal  $x$  `idealnrm(nf, x)`  
minimum of ideal  $x$  (direction  $v$ ) `idealmin(nf, x, v)`  
LLL-reduce the ideal  $x$  (direction  $v$ ) `idealred(nf, x, {v})`

### Ideal Operations

add ideals  $x$  and  $y$  `idealadd(nf, x, y)`  
multiply ideals  $x$  and  $y$  `idealmul(nf, x, y, flag)`  
intersection of ideals  $x$  and  $y$  `idealintersect(nf, x, y, flag)`  
 $n$ -th power of ideal  $x$  `idealpow(nf, x, n, flag)`  
inverse of ideal  $x$  `idealinu(nf, x)`  
divide ideal  $x$  by  $y$  `idealdiv(nf, x, y, flag)`  
Find  $(a, b) \in x \times y$ ,  $a + b = 1$  `idealaddtoone(nf, x, {y})`

### Primes and Multiplicative Structure

factor ideal  $x$  in  $nf$  `idealfactor(nf, x)`  
recover  $x$  from its factorization in  $nf$  `factorback(x, nf)`  
decomposition of prime  $p$  in  $nf$  `idealprimedec(nf, p)`  
valuation of  $x$  at prime ideal  $pr$  `idealval(nf, x, pr)`  
weak approximation theorem in  $nf$  `idealchinese(nf, x, y)`  
give  $bid$  = structure of  $(\mathbf{Z}_K/id)^*$  `idealstar(nf, id, flag)`  
discrete log of  $x$  in  $(\mathbf{Z}_K/bid)^*$  `ideallog(nf, x, bid)`  
`idealstar` of all ideals of norm  $\leq b$  `ideallist(nf, b, flag)`  
add archimedean places `ideallistarch(nf, b, {ar}, flag)`  
init `prmod` structure `nfmodprinit(nf, pr)`  
kernel of matrix  $M$  in  $(\mathbf{Z}_K/pr)^*$  `nfkernelpr(nf, M, prmod)`  
solve  $Mx = B$  in  $(\mathbf{Z}_K/pr)^*$  `nfsolvemodpr(nf, M, B, prmod)`

### Galois theory over $\mathbf{q}$

initializes a Galois group structure `galoisinit(pol, {den})`  
action of  $p$  in `nfgaloisconj` form `galoispermtopol(G, {p})`  
identifies as abstract group `galoisidentify(G)`  
exports a group for GAP or MAGMA `galoisexport(G, flag)`  
subgroups of the Galois group  $G$  `galoissubgroups(G)`  
subfields from subgroups of  $G$  `galoissubfields(G, flag, {v})`  
fixed field `galoisfixedfield(G, perm, flag, {v})`  
is  $G$  abelian? `galoisisabelian(G, flag)`  
abelian number fields `galoissubcyclo(N, H, flag, {v})`

### Relative Number Fields (rnf)

Extension  $L/K$  is defined by  $g \in K[x]$ . We have  $order \subset L$ .  
absolute equation of  $L$  `rnfequation(nf, g, flag)`  
relative `nfalgtobasis` `rnfalgtobasis(rnf, x)`  
relative `nfbasistoalg` `rnfbasistoalg(rnf, x)`  
relative `idealhnf` `rnfidealhnf(rnf, x)`  
relative `idealmul` `rnfidealmul(rnf, x, y)`  
relative `idealtwoelt` `rnfidealtwoelt(rnf, x)`

### Lifts and Push-downs

absolute  $\rightarrow$  relative repres. for  $x$  `rnfeltabstorel(rnf, x)`  
relative  $\rightarrow$  absolute repres. for  $x$  `rnfeltreltoabs(rnf, x)`  
lift  $x$  to the relative field `rnfeltup(rnf, x)`  
push  $x$  down to the base field `rnfeltdown(rnf, x)`  
idem for  $x$  ideal: (`rnfideal`)`reltoabs`, `abstorel`, `up`, `down`

### Projective $\mathbf{Z}_K$ -modules, maximal order

relative `polred` `rnfpolred(nf, g)`  
relative `polredabs` `rnfpolredabs(nf, g)`  
characteristic poly. of  $a \bmod g$  `rnfcharpoly(nf, g, a, {v})`  
relative Dedekind criterion, prime  $pr$  `rnfdedekind(nf, g, pr)`  
discriminant of relative extension `rnfdisc(nf, g)`  
pseudo-basis of  $\mathbf{Z}_L$  `rnfpseudobasis(nf, g)`  
relative HNF basis of  $order$  `rnfhnfbasis(bnf, order)`  
reduced basis for  $order$  `rnfillgram(nf, g, order)`  
determinant of pseudo-matrix  $A$  `rnfdet(nf, A)`  
Steinitz class of  $order$  `rnfsteynitz(nf, order)`  
is  $order$  a free  $\mathbf{Z}_K$ -module? `rnfisfree(bnf, order)`  
true basis of  $order$ , if it is free `rnfbasis(bnf, order)`

### Norms

absolute norm of ideal  $x$  `rnfidealnrmabs(rnf, x)`  
relative norm of ideal  $x$  `rnfidealnrmrel(rnf, x)`  
solutions of  $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$  `bnfisintnorm(bnf, x)`  
is  $x \in \mathbf{Q}$  a norm from  $K$ ? `bnfisnorm(bnf, x, flag)`  
initialize  $T$  for norm eq. solver `rnfisnorminit(K, pol, flag)`  
is  $a \in K$  a norm from  $L$ ? `rnfisnorm(T, a, flag)`

Based on an earlier version by Joseph H. Silverman  
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